Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2011

Marking Scheme

APPLIED MATHEMATICS

Ordinary Level

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. The points P and Q lie on a straight level road.
A car passes P with a speed of $10\ \text{m s}^{-1}$ and accelerates uniformly for 6 seconds to a speed of $22\ \text{m s}^{-1}$.
The car then decelerates uniformly to a speed of $18\ \text{m s}^{-1}$ and travels 80 m during this deceleration.
The car now maintains a constant speed of $18\ \text{m s}^{-1}$ for 3 seconds and then passes Q.
Find **(i)** the acceleration

(ii) the deceleration

(iii) $|PQ|$, the distance from P to Q

(iv) the average speed of the car, correct to one decimal place, as it moves from P to Q.

(i) $v = u + ft$

$22 = 10 + f(6)$

$f = 2\ \text{m s}^{-2}$ — 10

(ii) $v^2 = u^2 + 2fs$

$(18)^2 = (22)^2 + 2f(80)$

$f = -1\ \text{m s}^{-2}$ — 10

(iii) $s = ut + \frac{1}{2}at^2$

$s_1 = 10(6) + \frac{1}{2}(2)(36)$

$s_1 = 96\ \text{m.}$ — 10

$s_2 = 80\ \text{m}$

$s = ut + \frac{1}{2}at^2$

$s_3 = 18(3) + 0 = 54\ \text{m}$ — 5

$|PQ| = 96 + 80 + 54$

$= 230\ \text{m}$ — 5

(iv) $t_2 = \frac{v-u}{f} = \frac{18-22}{-1} = 4$ — 5

$v = \frac{s}{t}$

$= \frac{230}{6+4+3}$

$= 17.7\ \text{m s}^{-1}$ — 5

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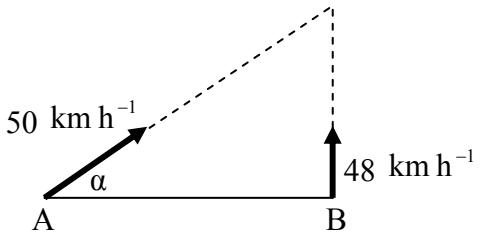
2. Ship A is 126 km due west of ship B.
A is moving at a constant speed of 50 km h^{-1}
in the direction east α north where $\tan\alpha = \frac{24}{7}$.



B is moving due north
at a constant speed of 48 km h^{-1}.

Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

Ship A intercepts ship B after *t* hours.

Find **(iv)** the value of *t*

(v) the distance each ship travels in this time *t*.

(i) $\vec{V}_A = 50\cos\alpha\,\vec{i} + 50\sin\alpha\,\vec{j}$ — 5
$= 14\,\vec{i} + 48\,\vec{j}$ — 5

(ii) $\vec{V}_B = 0\,\vec{i} + 48\,\vec{j}$ — 10

(iii) $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ — 5
$= (14\,\vec{i} + 48\,\vec{j}) - (0\,\vec{i} + 48\,\vec{j})$
$= 14\,\vec{i} + 0\,\vec{j}$ — 5

(iv) $t = \frac{126}{14}$
$= 9$ h — 10

(v) A $s = 50 \times 9$
$= 450$ km — 5

B $s = 48 \times 9$
$= 432$ km — 5

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3. A particle is projected from a point on horizontal ground with an initial speed of 58 m s^{-1} at an angle β to the horizontal, where $\tan\beta = \frac{20}{21}$.

(i) Find the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$.

(ii) Calculate the time taken to reach the maximum height.

(iii) Calculate the maximum height of the particle above ground level.

(iv) Find the range.

(v) Find the two times at which the height of the particle is 75 m.

(i)
$$\vec{u} = 58\cos\beta\,\vec{\text{i}} + 58\sin\beta\,\vec{\text{j}}$$
$$= 58\left(\frac{21}{29}\right)\vec{\text{i}} + 58\left(\frac{20}{29}\right)\vec{\text{j}}$$
$$= 42\,\vec{\text{i}} + 40\,\vec{\text{j}}$$
10

(ii)
$$v = u + f\,t$$
$$0 = 40 - 10\,(t)$$
$$t = 4\,\text{s}$$
10

(iii)
$$s_y = ut + \tfrac{1}{2}at^2$$
$$= 40(4) - 5(4)^2$$
$$= 80\ \text{m}$$
10

(iv)
$$s_x = ut + \tfrac{1}{2}at^2$$
$$= 42(8) + 0$$
$$= 336\ \text{m}$$
10

(v)
$$s_y = ut + \tfrac{1}{2}at^2$$
$$75 = 40t - 5t^2$$
$$0 = t^2 - 8t + 15$$
$$0 = (t-3)(t-5)$$
$$\Rightarrow t = 3,\ \ t = 5\,\text{s}$$
10

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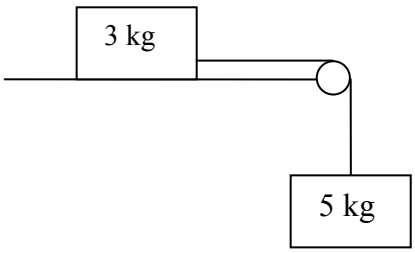
4. **(a)** A particle of mass 3 kg is connected to a particle of mass 5 kg by a taut, light, inextensible string which passes over a smooth light pulley at the edge of a rough horizontal table.

The coefficient of friction between the 3 kg mass and the table is $\frac{2}{3}$.



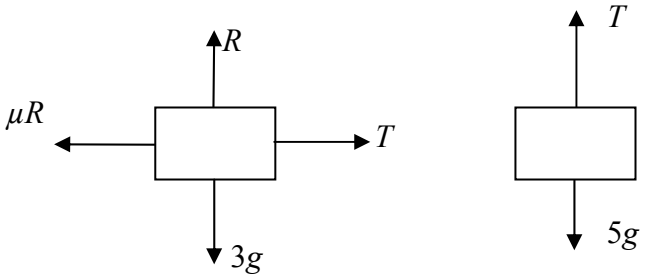
The system is released from rest.

(i) Show on separate diagrams the forces acting on each particle.

(ii) Find the common acceleration of the particles.

(iii) Find the tension in the string.

(i)



(ii)

$$\left.\begin{aligned} 5g - T &= 5a \\ T - \mu R &= 3a \end{aligned}\right\}$$

$$5g - \mu R = 8a$$
$$5g - 2g = 8a$$
$$a = \frac{3g}{8} \text{ or } \frac{15}{4} \text{ m s}^{-2}$$

(iii)

$$T = 5g - 5a$$
$$= 50 - \frac{75}{4}$$
$$= 31.25 \text{ N}$$

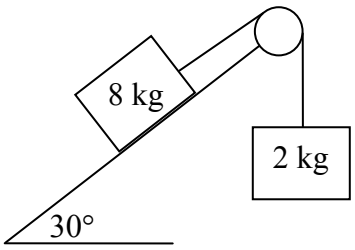
10 5 5 5 | 25

4 **(b)** Masses of 8 kg and 2 kg are connected by a light inelastic string which passes over a smooth light pulley as shown in the diagram.

The 8 kg mass lies on a smooth plane which is inclined at $30°$ to the horizontal.

The 2 kg mass hangs vertically.

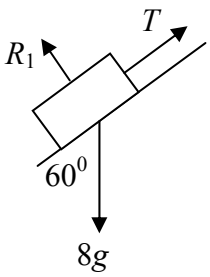
The system is released from rest.

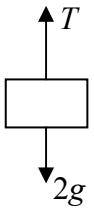


(i) Find the common acceleration of the masses.

(ii) Find the tension in the string.

(i)





$$T - 2g = 2a \quad (5)$$

$$8g\cos 60 - T = 8a \quad (5)$$

$$4g - 2g = 10a$$

$$20 = 10a$$

$$a = 2 \text{ m s}^{-2} \quad (5)$$

(ii)

$$T = 2g + 2a$$

$$= 24 \text{ N} \quad (5)$$

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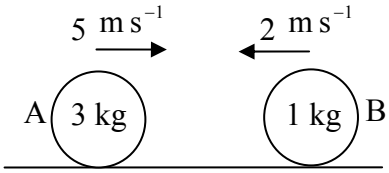
5. A smooth sphere A, of mass 3 kg, collides directly with another smooth sphere B, of mass 1 kg, on a smooth horizontal table.



Before impact A and B are moving in opposite directions with speeds of $5\ \text{m s}^{-1}$ and $2\ \text{m s}^{-1}$, respectively.

The coefficient of restitution for the collision is $\frac{1}{7}$.

Find **(i)** the speed of A and the speed of B after the collision

(ii) the loss in kinetic energy due to the collision

(iii) the magnitude of the impulse imparted to B due to the collision.

(i)
$$3(5) + 1(-2) = 3v_1 + 1(v_2)$$
$$13 = 3v_1 + v_2 \qquad \text{10}$$

$$v_1 - v_2 = -e(5+2)$$
$$= -\frac{1}{7}(7) \qquad \text{10}$$
$$= -1$$

$$v_1 = 3\ \text{m s}^{-1} \text{ and } v_2 = 4\ \text{m s}^{-1} \qquad \text{10}$$

(ii)
$$\text{KE}_b = \tfrac{1}{2}(3)(5)^2 + \tfrac{1}{2}(1)(-2)^2$$
$$= 39.5 \qquad \text{5}$$
$$\text{KE}_a = \tfrac{1}{2}(3)(3)^2 + \tfrac{1}{2}(1)(4)^2$$
$$= 21.5$$
$$\text{KE}_b - \text{KE}_a = 39.5 - 21.5$$
$$= 18\ \text{J} \qquad \text{5}$$

(iii)
$$I = |(1)(-2) - (1)(4)|$$
$$= 6\ \text{N s} \qquad \text{10}$$

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6. **(a)** Particles of weight 5 N, 8 N, 3 N and 1 N are placed at the points (4, 1), (–3, p), (p, q) and (15, 4), respectively.

The co-ordinates of the centre of gravity of the system are (1, p).

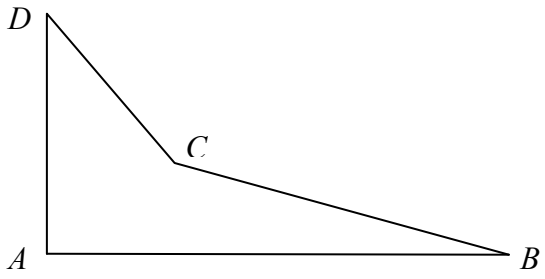
Find **(i)** the value of p

(ii) the value of q.

(b) A quadrilateral lamina has vertices A, B, C and D.

The co-ordinates of the vertices are $A(0, 0)$, $B(18, 0)$, $C(6, 3)$, $D(0, 9)$.

Find the co-ordinates of the centre of gravity of the lamina.



(a)

$$1 = \frac{5(4)+8(-3)+3(p)+1(15)}{17} \qquad 10$$

$$p = 2 \qquad 5$$

$$2 = \frac{5(1)+8(2)+3(q)+1(4)}{17} \qquad 10$$

$$q = 3 \qquad 5$$

(b)

ABC	$\frac{1}{2}(18)(3) = 27$	$(8,1)$	5
ACD	$\frac{1}{2}(9)(6) = 27$	$(2,4)$	5
$ABCD$	$= 54$	(x, y)	

$$(54)(x) = 27(8) + 27(2) \qquad 5$$

$$x = 5$$

$$(54)(y) = 27(1) + 27(4) \qquad 5$$

$$y = 2.5$$

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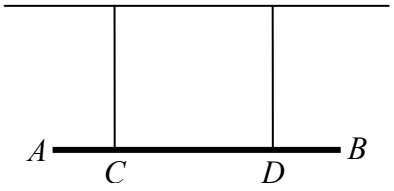
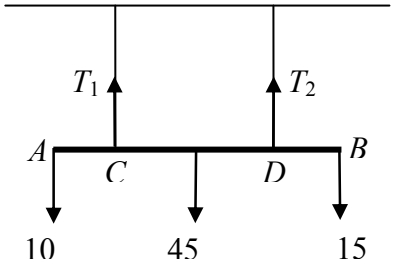
7. **(a)** A uniform beam, AB, is held in a horizontal position by two vertical inelastic strings attached at points C and D respectively.



The weight of the beam is 45 N.
The length of the beam is 2 m.

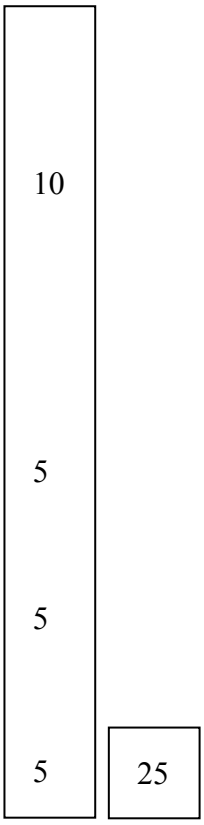
A particle of weight 10 N is attached at A and a particle of weight 15 N is attached at B.
$|AC| = |BD| = \frac{1}{2}$ m.
Calculate the tension in each of the strings.



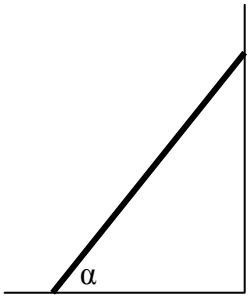
$$T_1 + T_2 = 10 + 45 + 15$$
$$= 70$$

$$T_1\left(\tfrac{1}{2}\right) + T_2\left(\tfrac{3}{2}\right) = 45(1) + 15(2)$$
$$T_1 + 3T_2 = 150$$

$$\Rightarrow T_1 = 30, \quad T_2 = 40$$



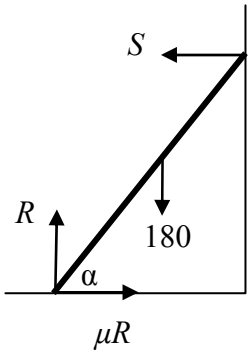
7 **(b)** A uniform ladder, of weight 180 N, rests on rough horizontal ground and leans against a smooth vertical wall.



The length of the ladder is 6 m and the angle between the ladder and the ground is α, where $\tan\alpha = \frac{12}{5}$.

The ladder is in equilibrium and is on the point of slipping.

Find the coefficient of friction between the ladder and the ground.



$\mu R = S$ 5

$R = 180$ 5

$S(6\sin\alpha) = 180(3\cos\alpha)$

$S\tan\alpha = 90$

$S\left(\frac{12}{5}\right) = 90$

$S = \frac{150}{4}$ 5

$\mu(180) = \frac{150}{4}$

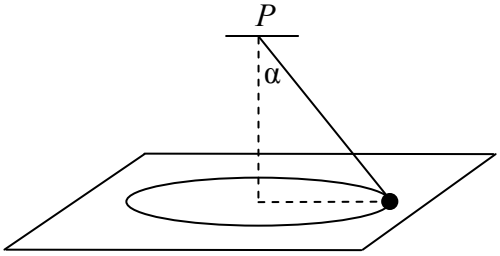
$\mu = \frac{5}{24}$ 5

25

8. **(a)** A particle describes a horizontal circle of radius 2 m with uniform angular velocity ω radians per second.
Its speed is 8 m s^{-1}.
Find **(i)** the acceleration of the particle

(ii) the time taken to complete one revolution.

(b) A smooth particle of mass 3 kg is attached by a light inelastic string to a fixed point P. The particle describes a horizontal circle of radius 0·5 m on the smooth surface of a horizontal table.



The centre of the circle is vertically below P.

The string makes an angle α with the vertical, where $\tan\alpha = \frac{4}{3}$.

The speed of the particle is 2 m s^{-1}.
Find **(i)** the tension in the string

(ii) the reaction force between the particle and the table.

(a)

(i) $v = r\omega$
$8 = 2\omega$
$\Rightarrow \omega = 4$ — 10

$a = r\omega^2$
$= 2(16)$
$= 32 \text{ m s}^{-2}$ — 10

(ii) $T = \frac{2\pi}{\omega}$
$= \frac{\pi}{2}$ s — 10

(b)

(i) $T\sin\alpha = \frac{mv^2}{r}$
$T\left(\frac{4}{5}\right) = \frac{3(4)}{\frac{1}{2}}$
$T = 30$ N — 10

(ii) $R + T\cos\alpha = 3g$
$R + 30\left(\frac{3}{5}\right) = 30 \Rightarrow R = 12$ N — 10

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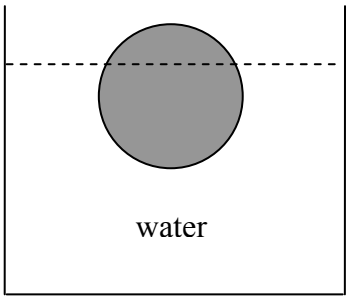
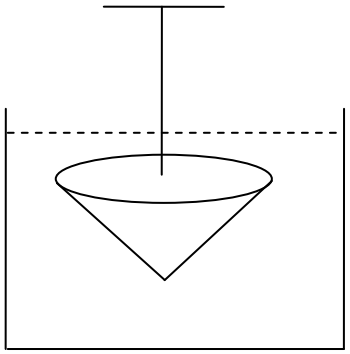
9. **(a)** A solid sphere, of radius 14 cm, floats at rest in water.
75% of the sphere lies below the surface of the water.
Find the weight of the sphere, correct to the nearest Newton.



(b) A solid cone of radius 10 cm and height 12 cm has relative density 7.
It is completely immersed in a liquid of relative density 0·9.
The cone is held at rest by a light inelastic vertical string which is tied to a fixed support.
The upper surface of the cone is horizontal.
Find the tension in the string, correct to the nearest Newton.

[Density of water = 1000 kg m^{-3}].



(a)

$$B = W$$

$$\frac{\frac{3}{4}W(1)}{s} = W$$

$$\Rightarrow s = 0.75$$ 10

$$W = \rho V g$$

$$W = 750\left\{\tfrac{4}{3}\pi(0.14)^3\right\}(10)$$

$$= 86 \text{ N}$$ 10

(b)

$$B = 900\left\{\tfrac{1}{3}\pi \times (0.1)^2 \times 0.12\right\}(10)$$ 10

$$= 3.6\pi$$

$$W = 7000\left\{\tfrac{1}{3}\pi \times (0.1)^2 \times (0.12)\right\}(10)$$ 10

$$= 28\pi$$

$$T + B = W$$

$$T = 28\pi - 3.6\pi$$

$$= 77 \text{ N}$$ 10 | 50